Transport and its enhancement caused by coupling

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Using the Weiss mean-field approximation theory and the particles' transport theory for the spatially periodic stochastic systems, we derive an exact analytical expression for the stationary probability current of a coupled lattice system driven by dichotomous noise. It is shown that, for this coupled lattice system, the spatial asymmetry of the system, the asymmetry of the dichotomous noise, and the coupling among nearest neighbors are the ingredients for the stationary probability current. By applying our theory to two special models, we find that (1) the coupling can lead to the directional transport of the particles (even when the potential and the dichotomous noise are symmetric) and (2) the coupling among nearest neighbors can enhance the transport of the particles in some circumstances. Our results are applied to a device of two-dimensional Josephson-junction arrays and a large protein motors cluster.

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I. INTRODUCTION

The nonequilibrium phenomena caused by noise in nonlinear systems have recently attracted a great deal of attention in a variety of contexts [1]. Generally, these phenomena involve a response of the system that is not only produced or enhanced by the presence of the noise, but also optimized for certain values of the parameters of the noise. One example of these phenomena is the phenomenon of stochastic resonance [2–4], wherein the response of a nonlinear system to a signal is enhanced by the presence of noise and maximized for certain values of the noise parameters or the input signal parameters. Another is the "Brownian motor," the one of interest to us in this paper, wherein intrinsically unbiased Brownian motion in stochastic spatial periodic potentials with spatial asymmetry or noise asymmetry leads to a systematic drift motion whose magnitude and even direction can be tuned by parameters of the noise [5]. Third is the nonequilibrium transition for the systems with finite or infinite coupled oscillators, which probably is a phase transition (the first order or second order) or not [6,7]. For these systems, the most exciting is that a re-entrant second-order phase transition has been found for a general spatially extended model by Van den Broeck and his collaborators [6]. The fourth such phenomenon is "resonant activation" which was first identified by Doering and Gadoua [8] and further studied by a number of other authors [9–13]. Here, the mean first passage time (MFPT) of a particle driven by noise over a fluctuating potential barrier exhibits a minimum as a function of the fluctuating potential barrier flipping rate, the noise transition rate, or the input signal frequency.

In this paper, we will investigate the transport of particles induced by dichotomous noise for a spatially periodic system with locally coupled oscillators. The transport of particles driven by dichotomous noise under a spatially periodic potential still attracts a great deal of attention (see Refs. [14–18] for examples and interesting results). In Ref. [14], Kula *et al.* studied the transport of particles caused by the additive dichotomous noise (in the absence of the thermal noise). Subsequently, in Ref. [15], they reported their investigation results for the transport of particles for the case with the additive dichotomous noise together with the thermal (Gaussian) noise. But they did not consider the case for the system with coupled oscillators driven by dichotomous noise (including additive dichotomous noise case and multiplicative dichotomous noise case). Later, Klumpp *et al.* investigated the noise-induced transport of two-coupled oscillators driven by the dichotomous noise and thermal noise [16]. In Ref. [17], Fogedby *et al.* also only studied the transport of two-coupled particles subject to the dichotomous noise together with thermal noise. The main difference between Refs. [16,17] is that the former studied a model with the multiplicative dichotomous noise, while the latter investigated a model with the additive dichotomous noise. In Ref. [18], the transport of the particles for a linear-chain-coupled system driven by multiplicative dichotomous noise was investigated by Igarashi *et al.*, for which the oscillators were locally coupled. But, up to now, the transport of particles caused by dichotomous noise for the mean-field coupled systems has been little known.

In this paper, we will study the transport of particles induced by dichotomous noise for a mean-field coupled system which is a lattice one, in which every oscillator is driven by multiplicative dichotomous noise and locally coupled with the nearest-neighbor oscillators by mean field (after using the Weiss mean-field approximation, see below).

It will be shown that, for this system, the spatial asymmetry, the asymmetry of the dichotomous noise, and the coupling among nearest neighbors are ingredients for the nonzero stationary probability current. By applying our theory proposed in this paper to two special models, it will represent that the coupling among nearest neighbors can lead to the directional transport of the particles (even when the potential and the dichotomous noise are symmetric) and in some circumstances, the coupling can enhance the transport of the particles (i.e., the coupled particles can move faster than the single ones). The setup of the paper is arranged as follows. We will first present the general model of the system and derive its master equation in Sec. II. Then, in Sec. III, an exact analytical expression for the stationary probability current will be derived (some discussions will be given in this section). After that, using our theory, in Sec. IV, two special models will be investigated. Afterwards, in Sec. V, we will connect our results to two real systems, i.e., a device of two-dimensional (2D) Josephson-junction arrays and a large protein motors cluster. Finally, some conclusions and discussions will be given in Sec. VI.

II. GENERAL MODEL AND ITS MASTER EQUATION

We consider the lattice model with a set of scalar variables $\{x_i\}$, in which x_i is the variable defined on lattice point $i(i=1,2,\ldots,L^h)$ of a lattice in h dimensions. The equations of the Brownian particles for the system considered by us in the paper are (in the dimensionless form and in the Stratonovich case)

$$\dot{x}_i = f(x_i) + g(x_i)\xi_i(t) - \frac{D'}{2h}\sum_j (x_i - x_j),$$

$$f(x_i) = -\frac{d}{dx_i}U_1(x_i), \quad g(x_i) = -\frac{d}{dx_i}U_2(x_i), \tag{1}$$

where the sum over j runs over the set of $2h$ nearest neighbors of site i, the system is only in one period, i.e., $nT+c \leq x_i \leq nT+d$ ($n=0,1,2,3,\ldots$ and the period $T=d-c$), $U_1(x_i)$ and $U_2(x_i)$ are spatially periodic functions of x_i with period $T=d-c$, D' is a coupling constant, and $\xi_i(t)$ is a zero-mean dichotomous noise which takes values $-a$ and b ($a,b>0$) whose transition rate from $-a$ to b is γ_1 and vice versa is γ_2. Here, we present the Weiss mean-field approximation

$$s \doteq \langle x \rangle = F(s). \tag{2}$$

In the literature, the above equation is called the Weiss mean-field approximation [19], which has been extensively applied [6,7,20], although it may happen that this approximation was first published in Ref. [19]. In this approximation, all the oscillators have an identical evolution given by the following stochastic equation with the mean field s:

$$\dot{x} = f(x) + g(x)\xi(t) - D'(x-s). \tag{3}$$

The master equations for the probability density distribution of Eq. (3) are [21]

$$\partial_t P^+(x,-a,s,t) = -\partial_x[f(x) - ag(x) - D'x + D's]P^+(x,-a,s,t) + \gamma_2 P^-(x,b,s,t) - \gamma_1 P^+(x,-a,s,t), \tag{4}$$

$$\partial_t P^-(x,b,s,t) = -\partial_x[f(x) + bg(x) - D'x + D's]P^-(x,b,s,t) + \gamma_1 P^+(x,-a,s,t) - \gamma_2 P^-(x,b,s,t). \tag{5}$$

Let $P(x,s,t)=P^+(x,-a,s,t)+P^-(x,b,s,t)$ and $w(x,s,t)=-aP^+(x,-a,s,t)+bP^-(x,b,s,t)$, where $P(x,s,t)$ is the probability density of the system studied by us. From Eqs. (4) and (5), we can get

$$\partial_t P(x,s,t) = -\partial_x[f(x) - D'x + D's]P(x,s,t) - \partial_x g(x)w(x,s,t) = -\partial_x J(x,s,t), \tag{6}$$

$$\partial_t w(x,s,t) = -\partial_x[f(x) - D'x + D's + \theta]w(x,s,t) - \frac{D}{\tau}\partial_x g(x)P(x,s,t) - \frac{1}{\tau}w(x,s,t), \tag{7}$$

with the probability current

$$J(x,s,t) = [f(x) - D'x + D's]P(x,s,t) + g(x)w(x,s,t), \tag{8}$$

where $D=ab/\tau$ is the noise's strength, $\tau=1/(\gamma_1+\gamma_2)$ is the correlation time of the noise, and $\theta=b-a$ which is defined as the asymmetry parameter of the dichotomous noise [14,22].

Now, what we are interested in is the stationary probability current. In the next section, we will derive it.

III. STATIONARY PROBABILITY CURRENT

In the limit of $t\to\infty$, i.e., for the stationary state, $P(x,s,t)\to P(x,s)$ and $J(x,s,t)\to J=\text{const}$. Then from Eqs. (7) and (8), we have

$$J = [f(x) - D'x + D's]P(x,s) + g(x)w(x,s), \tag{9}$$

$$\partial_x[f(x) - D'x + D's + \theta]w(x,s) + \frac{D}{\tau}\partial_x g(x)P(x,s) + \frac{1}{\tau}w(x,s) = 0. \tag{10}$$

From Eq. (9), we can get

$$w(x,s) = \frac{J}{g(x)} - \frac{f(x) - D'x + D's}{g(x)}P(x,s). \tag{11}$$

Substituting Eq. (11) into Eq. (10), one obtains

$$A(x,s)P(x,s) - \partial_x B(x,s)P(x,s) = JC(x), \tag{12}$$

where $A(x,s)=[f(x)-D'(x-s)]/[\tau g(x)]$, $B(x,s)=-[f(x)-D'(x-s)+\theta][f(x)-D'(x-s)]/g(x)+Dg(x)/\tau$, and $C(x,s)=1/[\tau g(x)]+d\{[f(x)-D'(x-s)+\theta]/g(x)\}/dx$.

For convenience, we define

$$\phi(x,s) = \int_{c+nT}^{x+nT}\frac{A(x',s)}{B(x',s)}dx', \tag{13}$$

in which $n=1,2,3,\ldots$ and T is the period of U_1 and U_2. Then, dividing both sides of Eq. (12) by $e^{\phi(x,s)}$ and noting $\partial_x\phi(x,s)=A(x,s)/B(x,s)$, we can get [5]

$$-\partial_x\frac{B(x,s)P(x,s)}{e^{\phi(x,s)}} = J\frac{C(x,s)}{e^{\phi(x,s)}}. \tag{14}$$

Integrating Eq. (14) from $c+nT$ to $d+nT$, one gets

$$J = N\left[1 - \left(1 + \frac{\Delta}{B(c,s)}\right)e^{-\phi(d,s)}\right], \tag{15}$$

where $N=B(c,s)P(c,s)/\int_c^d C(x,s)e^{-\phi(x,s)}dx$, which is the normalization constant for the stationary probability distribution [cf. Eq. (16)], and $\Delta=D'T[2f(c)-2D'(c-s)+\theta]/g(c)-(D'T)^2/g(c)$.

Substituting Eq. (15) into Eq. (14) and integrating it from $c+nT$ to $x+nT$, one can get

$$P(x,s) = N\frac{e^{\phi(x,s)}}{B(x,s)}\left[\oint C(x',s)e^{-\phi(x',s)-\theta'(x-x')\phi(d,s)}dx' + \frac{\Delta}{B(c,s)}\int_{c+nT}^{x+nT} C(x',s)e^{-\phi(d,s)-\phi(x',s)}dx'\right], \quad (16)$$

where $\theta'(x-x')$ is the Heaviside step function and N the normalization constant.

In terms of the Weiss mean-field approximation, we neglect the fluctuation in the neighboring sites. Then, from Eqs. (2) and (16), we can get [6,7,19]

$$s \doteq \oint xP(x,s)dx, \quad (17)$$

which is a self-consistency equation whose solution yields the dependence of s on the other parameters. Then, from Eqs. (15)–(17), we can determine the stationary probability current of Eq. (1) or Eq. (3).

In what follows, we present both general and particular solutions for the stationary probability current J. From Eq. (15), we can find that the condition for the nonzero stationary probability current (i.e., $J \neq 0$) is

$$\left(1 + \frac{\Delta}{B(c,s)}\right)e^{-\phi(d,s)} \neq 1, \quad (18)$$

namely,

$$\int_{c+nT}^{d+nT} dx \frac{f(x) - D'(x-s)}{-\tau[f(x) - D'(x-s) + \theta][f(x) - D'(x-s)] + Dg(x)^2} \neq \ln\left(1 + \frac{\Delta}{B(c,s)}\right). \quad (19)$$

First, let us consider the case in Ref. [14], in which Kula *et al.* studied the transport of particles caused by the additive dichotomous noise. When $D'=0$ and $g(x)=1$, Eq. (3) becomes the model investigated in Ref. [14]. In this case, Eq. (19) becomes

$$\int_{c+nT}^{d+nT} \frac{f(x)}{-[f(x)+\theta]f(x) + D}dx \neq 0. \quad (20)$$

From Eq. (20), we can find that the nonzero stationary probability current can be gotten only when spatially periodic potential $U_1(x)$ is asymmetric or the dichotomous is asymmetric (i.e., $\theta \neq 0$). If the spatially periodic potential and the dichotomous noise are both symmetric [suitably selecting c and d, we have $U_1(c+d-x)=U_1(x)$, i.e., $f(c+d-x)=-f(x)$], the left-hand side of Eq. (20) equals zero, which can be easily obtained after using $f(c+d-x)=-f(x)$. Then, Eq. (20) does not hold and the stationary probability current J equals zero. In this case, our result accords with the one in Ref. [14].

Second, let us consider the multiplicative noise case without coupling (i.e., $D'=0$). Now, Eq. (19) reads

$$\int_{c+nT}^{d+nT} dx \frac{f(x)}{-\tau[f(x)+\theta]f(x) + Dg(x)^2} \neq 0. \quad (21)$$

From Eq. (21), it can be found that the nonzero stationary probability current appears in the case when $U_1(x)$ is asymmetric, $U_2(x)$ is asymmetric, or the dichotomous noise is asymmetric. If $U_1(x)$ and $U_2(x)$ are symmetric simultaneously together with $a=b$ (i.e., the dichotomous is symmetric) [suitably selecting c and d, we have $U_1(c+d-x)=U_1(x)$ and $U_2(c+d-x)=U_2(x)$, i.e., $f(c+d-x)=-f(x)$ and $g(c+d-x)=-g(x)$], now Eq. (21) is false and the stationary probability current equals zero [this can be easily gotten after using $f(c+d-x)=-f(x)$ and $g(c+d-x)=-g(x)$].

Finally, let us consider the multiplicative noise case with $D' \neq 0$. Now, from Eq. (19), we can observe that the asymmetries of U_1 and U_2, the asymmetry of the dichotomous noise (i.e., the nonzero value of $\theta=a-b$), and the presence of the coupling among the nearest neighbors (in the presence of the mean field) are all ingredients for the nonzero stationary probability current. If U_1 and U_2 are symmetric, $a=b$, and $D'=0$, no nonzero current emerges since the symmetry of the system cannot be broken. Here, we should note a special case, i.e., the one with U_1 and U_2 being symmetric and the dichotomous noise being symmetric but $D' \neq 0$ (or the additive noise case with U_1 being symmetric and $a=b$ but $D' \neq 0$). In this case, though U_1 and U_2 are symmetric (or U_1 is symmetric for the additive noise case) and the dichotomous noise is symmetric, the presence of the coupling can produce symmetry breaking of the system and therefore a nonzero stationary probability current can be caused (see Sec. IV B in the following).

IV. SPECIAL MODELS

In this section, using our above theory, we will investigate two special models. One is for the case with $U_1(x)$ and $U_2(x)$ being asymmetric, the dichotomous being asymmetric, and $D' \neq 0$. The other is for the additive noise case [i.e., $g(x) \equiv 1$] with $U_1(x)$ being symmetric and the noise being symmetric but $D' \neq 0$.

A. Model I

For this special model, $U_1(x)$, $U_2(x)$, and the dichotomous noise are all asymmetric, and coupling constant does not equal zero. For convenience and simplicity, here we take $U_1(x)$ and $U_2(x)$ to be piecewise linear (see Fig. 1). Then [cf. Fig. 1] when $-0.5 \leq x \leq k$, we can get

$$A(x,s) = [-2/(2k+1) - D'(x-s)]/\tau,$$

$$B(x,s) = D/\tau - [-2/(2k+1) - D'(x-s) + \theta][-2/(2k+1) - D'(x-s)],$$

$$C(x,s) = 1/\tau - D', \quad (22)$$

and when $k \leq x \leq 0.5$, we can get

$$A(x,s) = [2/(1-2k) - D'(x-s)]/\tau,$$

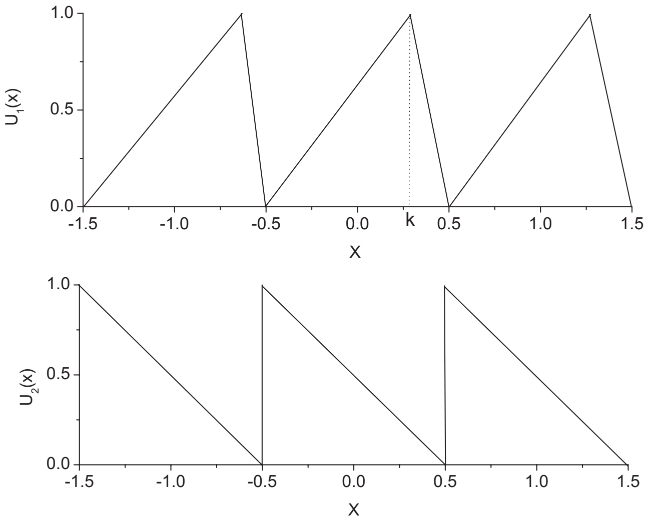

FIG. 1. $U_1(x)$ and $U_2(x)$ for model I.

$$B(x,s) = D/\tau - [2/(1-2k) - D'(x-s) + \theta][2/(1-2k) - D'(x-s)],$$

$$C(x,s) = 1/\tau - D'. \tag{23}$$

The stationary probability current can be gotten by substituting Eqs. (22) and (23) into the formulas (13) and (15)–(17).

For this model, according to the above analyses in Sec. III, we know that three aspects (i.e., the asymmetry of the potential, the asymmetry of the dichotomous noise, and the presence of the coupling among nearest neighbors) are ingredients for the nonzero stationary probability current. Now, every ingredient can cause the symmetry breaking of the system and therefore produce a nonzero current in the stationary state. Based on Eqs. (13), (15)–(17), (22), and (23), in Fig. 2 we plot some results of the stationary probability current J versus the noise's strength D for different values of the coupling (D'=0, 0.5, 1, 2, 3, 5, and 7, respectively) with the other parameters k=0.3, τ=0.01, and θ=−0.7. From this figure, we can get some characteristic features for the transport of particles of model I. First, we can observe that, for the parameters selected by us in Fig. 2, there are some peaks and wells to appear for the stationary probability currents versus the noise's strengths. We calculated a lot of cases for the currents as the function of the noise's strengths between D'=2 to D'=3 (such as D'=2.205, D'=2.21, D'=2.215, D'=2.22, D'=2.225, D'=2.23, and so on) with the same other parameters as in Fig. 2. We found that, there is a critical value D'_0 of the coupling constant, when $D' < D'_0$, a peak can emerge for the current versus the noise's strength, while when $D' > D'_0$, a well can appear for the current versus the noise's strength. For the selected parameters in Fig. 2, the critical value D'_0 approximately equals 2.2. Second, for the selected parameters in Fig. 2, if the coupling constant is larger than the critical value D'_0, by controlling the noise strength, a current reversal can be gotten (we can see the appearance of current reversal for the curves of D'=3, 5, and 7 in Fig. 2). Third, we can find that, for the fixed parameters in Fig. 2, there is a small region for the noise strength in which the current curves intersect. For the fixed parameters in Fig. 2, this small region is from $D_1 \approx 2.3$ to $D_2 \approx 2.85$. When $D > D_2$, the coupling among the nearest neighbors can enhance the transport of particles (i.e., with increasing the value of the coupling constant, the current can be increased), while when $D < D_1$, the coupling among the nearest neighbors can weaken the transport of particles (i.e., with the increase of the value of the coupling constant, the current can be decreased), in case that the current reversal does not happen (if the current reversal happens, our study shows that, with increasing the coupling constant, the negative transport can also be increased, see curves of D'=3, 5, and 7 in Fig. 2).

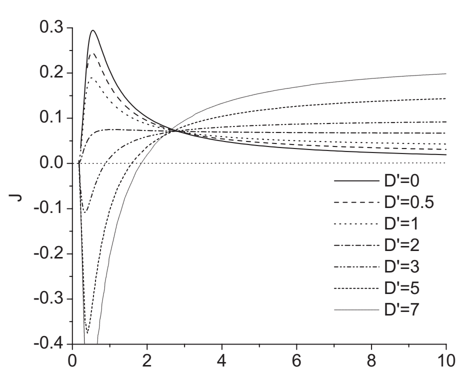

FIG. 2. Probability current J vs the noise strength D for different values of the coupling (D'=0, 0.1, 0.5, and 1, respectively) with the other parameters k=0.3, τ=0.01, and θ=−0.7 for model I.

Some analyses for Fig. 2 are given below. The sign of the stationary current J for smaller values of D as a function of D' depends on the shape of the potential. For a potential (U_1 plus U_2) as in Fig. 1, in the case of larger values of the coupling, it is simpler to jump to the left than to the right. Then, the stationary flux (i.e., the stationary current) moves in the direction from the right to the left and the value of J is negative (which can be seen in Fig. 2). The reason is that for a larger coupling plus a littler noise, the distance to reach the potential peak is more important than the maximal force needed (i.e., the slope of the potential). For smaller values of the coupling and still smaller noise, the slope of the potential is more important than the distance to reach the peak of the potential. This leads to the shift in the direction of the stationary flux from the left to the right and the stationary probability current J is positive (which can be observed from Fig. 2). Also for Fig. 2, for large D, the coupling always increases the flux (yet, the value of $|J|$ is always smaller than at the extremum point). As a result, the stationary flux originated from the coupling (with a littler noise) reflects more different physics than the stationary flux originated from the noise with some coupling. At the point $D=D_0$, a transition occurs between the two regimes.

The nonzero stationary probability current J for model I is produced by the nonzero values of D', θ, and k (i.e., the coupling constant among nearest neighbors, the asymmetry degree of the dichotomous noise, and the asymmetry degree of the potential) and has a *nonlinear* complicated dependence on D', θ, and k. To determine the *nonlinear* relationship between J and D', θ and k, one should make a lot of numerical simulations relied on Eqs. (15)–(17) for the model I. But, we believe that it is unnecessary to do this work since Eqs. (15)–(17) can give the nonlinear complex dependence for J on D', θ and k, and our main purpose of the model I is to show that the stationary probability current can be gotten when D', θ, and k are nonzero.

B. Model II

In this section, we consider a special model with $g(x) \equiv 1$ (i.e., the additive noise case), U_1 being symmetric, and the dichotomous noise being symmetric (i.e., $\theta = b - a = 0$), but $D' \neq 0$. We take $U_1(x)$ as the one in Fig. 1 but with $k = 0$. Then, we have

$$\left.\begin{aligned} A(x,s) &= [-2 - D'(x-s)]/\tau, \\ B(x,s) &= D/\tau - [-2 - D'(x-s)][-2 - D'(x-s)], \\ C(x,s) &= 1/\tau - D', \end{aligned}\right\} \quad \text{when} \quad -0.5 \le x \le 0, \tag{24}$$

$$\left.\begin{aligned} A(x,s) &= [2 - D'(x-s)]/\tau, \\ B(x,s) &= D/\tau - [2 - D'(x-s)][2 - D'(x-s)], \\ C(x,s) &= 1/\tau - D', \end{aligned}\right\} \quad \text{when} \quad 0 \le x \le 0.5. \tag{25}$$

Then, substituting Eqs. (24) and (25) into the formulas (13) and (15)–(17), we can calculate the stationary probability current.

For model II, as we analyzed in Sec. III, though the potential and the dichotomous noise are both symmetric, the presence of the coupling among nearest neighbors can induce the symmetry breaking and therefore cause the nonzero flux of the particles (in the stationary state). In Fig. 3(a), we plot the stationary probability current J as a function of the noise strength D for different values of D' (D'=0.1, 0.5, and 1, respectively) with the other parameter τ=0.01 (now k=0 and θ=0). This figure represents the appearance of the stationary probability current induced by symmetry breaking caused by the coupling among nearest neighbors. With the absence of the coupling (i.e., D'=0), no current can be produced (since now no symmetry is broken). In addition, from the figure, we can see that, for the parameters selected by us in this figure, with increasing the noise strength (maintaining the coupling to be unchanged), the current can be increased, and when $D \to \infty$, it tends to a saturated value J_0. [When D' equals 0.1, J_0 is about 0.004; when D' equals 0.5, J_0 is about 0.02; and when D' equals 0.1, J_0 is about 0.04. These points are marked by us in Fig. 3(a).] Moreover, an important thing is that, for this model in Fig. 3(a), our large number of numerical simulations shows that the coupling can enhance the transport of the particles (i.e., the increase of the value of the coupling constant with the other parameters maintaining unchanged can lead to the increase of the stationary probability current), which can be observed from Fig. 3(a). Figure 3(b) is the representation plotted for J/J_0 versus D with the same parameters as the ones selected by us in Fig. 3(a).

Here, we must mention that Figs. 3(a) and 3(b) show that the stationary probability current seems monotonous to depend on the noise's strength D (note that θ=0 and k=0). But this is only for the parameters selected by us in Figs. 3(a) and 3(b). Actually, J nonlinearly depends on all the parameters as shown by Eqs. (15)–(17). To illustrate this point of view, in Fig. 4, we plot J versus D with D'=0.5 and τ=1.4. Figure 4 can represent the nonlinear dependence of J on D, which is nonmonotonous. In addition, for the parameters selected in Fig. 4, there is an odd point in the vicinity of $D \approx 5.75$, which is due to the result of the zero denominator in Eqs. (15)–(17) for the selected parameters D'=0.5 and τ=1.4.

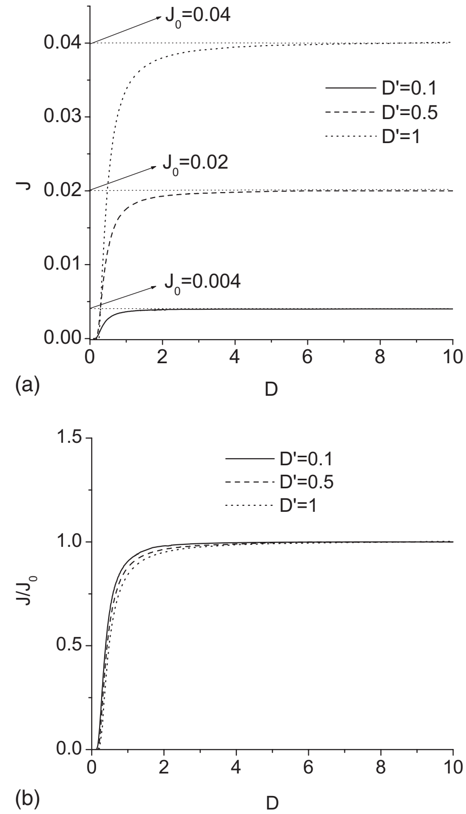

FIG. 3. (a) Probability current J as a function of the noise strength D for different values of D' (D'=0.1, 0.5, and 1, respectively) with the other parameter τ=0.01 (now k=0 and θ=0) for model II. (b) J/J_0 vs D for model II with the same parameters as in (a).

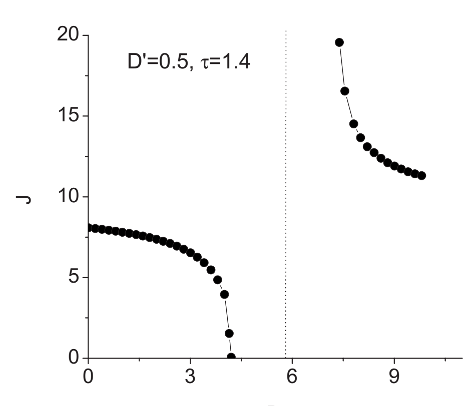

FIG. 4. J vs D with D'=0.5 and τ=1.4 for model II.

V. CONNECTION TO REAL SYSTEMS

In this section, we apply our results to two practical examples whose dynamics satisfies Eq. (1). One is a device of 2D Josephson-junction arrays in Refs. [23,24]. The other is a large protein motors cluster.

A. Device of 2D Josephson-junction arrays

We consider a device of 2D Josephson-junction arrays in Refs. [23,24]. Here, we assume that (1) for every site i of the lattices, the number of its nearest neighbors is large enough so that we can use the approximate mean-field theory [6,7,19] and (2) for every Josephson junction, its insulating material thickness L_i satisfies $L_i \ll \lambda_i$, where λ_i is the coherence length for the electron pairs to move into the insulating material [which can be gotten by the following method: the external electric current added to the arrays and the following current $\eta(t)-I_0$ added to every site i should be small enough in comparison to $(\hbar/(2eR))\langle d\phi_i/dt\rangle_s$, see Eq. (27) and Ref. [25]], so that we can get the approximate relationship $\sin(\phi_i-\phi_j)\approx\phi_i-\phi_j$ in which ϕ_i is the phase for the site i of the 2D Josephson-junction arrays in Refs. [22,23]. Then, according to Refs. [23,24,26], in the time-dependent Ginzburg-Landau (TDGL) dynamics [26], we have (not considering the magnetic field)

$$\hbar\frac{d\phi_i}{dt}\approx-\Gamma J_0\sum_j(\phi_i-\phi_j)+I_{ext}, \tag{26}$$

where Γ is a dimensionless constant which determines the time scale of relaxation and J_0 denotes the strength of the Josephson coupling between the site i and the nearest-neighbor sites j. The summation in Eq. (26) is over all nearest neighbors.

Afterwards, we put every site of the lattices into a fluctuating dichotomous electric current $\eta(t)$, which can be gotten by using the devices in Refs. [27,28] and a constant electric current $-I_0$. Then, Eq. (26) becomes (neglecting the thermal noise)

$$\hbar\frac{d\phi_i}{dt}\doteq-\Gamma J_0\sum_j(\phi_i-\phi_j)+\eta(t)-I_0, \tag{27}$$

in which $\eta(t)$ takes values c and d ($c>d>0$), the transition rate from c to d is λ_1 and vice versa is λ_2, and $I_0=(1/2)(c+d)$.

To correspond to Eq. (1), we assume that $\lambda_1=\lambda_2=\lambda$ and make the following transformation. Set $\eta(t)=g+\zeta(t)$, where g is a constant, $\zeta(t)$ is a dichotomous noise, which takes values E and $-E$ ($E>0$), and the transition rates from E to $-E$ for $\zeta(t)$ and vice versa are both λ. Using the relations between $\eta(t)$ and $\zeta(t)$, we can get $g=(1/2)(c+d)$ and $E=(1/2)(c-d)$. Taking ensembles average of $\eta(t)=g+\zeta(t)$, we can obtain $\langle\zeta(t)\rangle=0$. We can also derive the correlation function of $\zeta(t)$: $\langle\zeta(t)\zeta(t')\rangle=(E^2/\tau)\exp[-|t-t'|/\tau]$, with $\tau=1/(2\lambda)$.

Substituting $\eta(t)=g+\zeta(t)$ into Eq. (27), we get

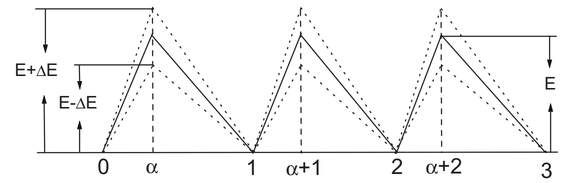

FIG. 5. Fluctuating potential barrier of the proteins motor.

$$\hbar\frac{d\phi_i}{dt}\doteq-\Gamma J_0\sum_j(\phi_i-\phi_j)+\zeta(t). \tag{28}$$

Equation (28) corresponds to the additive noise case of Eq. (1) with $f(x_i)=0$. So, according to our analyses in Sec. III and the results for the model II in Sec. IV, we can conclude that (1) we can get nonzero values of the voltage $\langle V_i\rangle_s=(\hbar/(2e))\langle d\phi_i/dt\rangle_s$ for every column in the 2D Josephson-junction arrays, even though the dichotomous noise $\zeta(t)$ is symmetric [since $\langle d\phi_i/dt\rangle_s\neq 0$ [24], which can be easily seen from Eq. (19)], and (2) the coupling can enhance the voltage $\langle V_i\rangle_s$ in some circumstances.

B. Large protein motors cluster

For the proteins motor, when it moves along a biopolymer (which is a linear highway with a periodic array of fixed charges), its dynamic equation for the position satisfies (if we do not consider the thermal noise) [29]

$$\frac{dx}{dt}=-\partial_x U(x,t), \tag{29}$$

where $U(x,t)$ is a fluctuating potential barrier, which is caused by the repeated binding of ATP and release of ADP when a protein motor moves along the biopolymer.

In Fig. 5, we plot the fluctuating potential barrier $U(x,t)$ (see Ref. [29]). We can observe that the force $F=-\partial_x U(x,t)$ fluctuates between $F_1^+=-(E+\Delta E)/\alpha$ and $F_1^-=-(E-\Delta E)/\alpha$ on the interval $(0,\alpha)$ and between $F_2^+=(E+\Delta E)/(1-\alpha)$ and $F_2^-=(E-\Delta E)/(1-\alpha)$ on the interval $(\alpha,1)$. The flipping rate of the fluctuating potential barrier is γ.

The force $F=-\partial_x U(x,t)$ can be rewritten as the following form:

$$-\partial_x U(x,t)=-\partial_x U(x)+\eta_0(t), \tag{30}$$

in which $U(x)$ is the solid lines barrier depicted in Fig. 5 [i.e., the determined asymmetric potential barrier (which is ratchet)], $\eta_0(t)$ is a dichotomous noise which takes values ΔE and $-\Delta E$, and the transition rates for $\eta_0(t)$ from ΔE to $-\Delta E$ and vice versa are γ. Substituting Eq. (30) into Eq. (29), we get

$$\frac{dx}{dt}=w(x)+\eta_0(t), \tag{31}$$

where $w(x)=-\partial_x U(x)$.

Below, we consider a large protein motors cluster. We assume that it is composed of n single protein motors and n is large enough so that we can use the approximate mean-field theory [6,7,19]. Similarly, as in Refs. [30–32], we assume that only nearest motors can be coupled and the coupling force F_0 between the motor i and the motor j satisfies $F_0 = -D_0(x_i - x_j)$ (the motor i and the motor j are the nearest ones) [31]. Then, we have

$$\frac{dx_i}{dt} = w(x_i) + \eta_0(t) - D_0 \sum_j (x_i - x_j), \tag{32}$$

where the summation is over all nearest neighbors and D_0 is the coupling constant.

Equation (32) can correspond to Eq. (1). So, according to our above analyses in Sec. III, we can conclude that the asymmetry of the potential $U(x)$ and the coupling among the nearest motors can break the symmetry of the system and therefore produce a nonzero flux (i.e., the stationary probability current) of the protein motors, even though the dichotomous noise is symmetry. A special case for Eq. (32) is that the potential $U(x)$ is also symmetry. Now, according to our results in Sec. IV for the model II, we can conclude that the coupling among the nearest motors cannot only produce a nonzero flux of the protein motors but also enhance the movement of the proteins motors in some circumstances (i.e., increasing the coupling can lead to the increase of the average velocity of the proteins motors and the coupled protein motors can move faster that the single one), which accords with the theoretical results in Ref. [30] and the experimental results in Ref. [32] for the coupled protein motors (that the coupled motors can move fast than the single one in some circumstances).

Up to now, the protein motor clusters have been studied little (so they have been known little), especially the interaction between the nearest neighbors of these motors. Generally, the interaction between the nearest neighbors is regarded as the one in Refs. [30,31] and in Eq. (32). If it is regarded as the state electric one in which the interaction force among the nearest neighbors i and j is proportional to $(x_i - x_j)^{-2}$, the theoretical results will not accord with the experimental ones [30–32]. Of course, it remains to be further studied if this interaction is a long-distance one or a short-distance one experimentally.

VI. CONCLUSIONS AND DISCUSSIONS

In conclusion, in this paper, by applying our theory (see Sec. III or see below) to two special models (see Sec. IV), we have shown that (1) the coupling among nearest neighbors can lead to the directional transport of the particles [even sometimes in a perfectly symmetric system (i.e., the potential and the dichotomous noise are both symmetric)] and (2) the coupling can enhance the transport of the particles (i.e., the coupled particles can move faster than the single ones) in some circumstances. Using the Weiss mean-field approximation [6,7,19] and the particles' transport theory for the spatially periodic stochastic systems proposed in Refs. [5,21], we have derived the exact analytical expression for the stationary probability current for the coupled lattice system. The theory proposed by us is that, for the coupled lattice system in the present paper, the spatial asymmetry of the system, the asymmetry of the dichotomous noise, and the coupling among different nearest neighbors are the ingredients for the stationary probability current. Now, every ingredient can break the symmetry of the system and therefore can produce the transport of particles. In the presence of the spatial symmetry of the system, the symmetry of the dichotomous noise, and the zero value of the coupling constant, no stationary probability current can be caused (since no symmetry-breaking appears). In addition, our results are applied to a device of 2D Josephson-junction arrays and a large protein motors cluster.

Finally, in this paper, we did not consider the thermal (Gaussian) noise (white and not colored) [33]. If we add a thermal (Gaussian white) noise in Eq. (1) [34], the above method for the derivation of the exact analytical expression of the stationary probability current of the system will not be applicable, even the spatial functions $U_1(x)$ and $U_2(x)$ are piecewise linear. Besides, the other author's methods, such as Kula *et al.* [14,15], Igarashi *et al.* [18], Kolomeisky *et al.* [30], and so on, are also void for the derivation of the analytical expression of the stationary probability current (exact or approximate) of the system, owing to the complexity induced by the coupling part in Eq. (1). So, it makes us avoid the complexity (produced by the coupling part) and the consequent impossibility of the derivation of the exact analytical expression of the s stationary probability current that we do not consider the thermal noise. All the same, the results that the coupling among the nearest neighbors can produce and enhance the transport of the particles can be gotten by us by applying our theory to the two special models. Of course, it remains to be proved if the coupling can cause and enhance the transport of the particles in other mean-field-coupled models, except for our two special models in the paper.

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